Module
9

DC Machines

Lesson
41

Problem Solving on D.C Machines

Contents

Introduction

In this lecture some typical problems on D.C machines are worked out not only to solve the problems only but also to bring out important features of the motors involving its performance. To begin with few problems on d.c motors have been solved and then problems on generator are taken up.

For a beginner, it is suggested to follow the following guidelines for successful completion of the problem.

1. Read the problem and note down the informations provided about the motor.

2. In general, some initial steady operating conditions are given in terms of armature current, field current, speed etc.

3. Draw a circuit diagram showing the initial variable values.

4. Write down the *back emf* equation and the *torque* equation for the initial operating point, it is reiterated here that these two equations hold the key for the gateway of correct solution.

5. In the statement of the problem, with respect to the initial operating point, some variable (like, armature resistance, field resistance, load torque, armature applied voltage etc.) will be changed and you will be asked perhaps to calculate new armature current, speed etc.

6. Assume the variables to be calculated and write down once again the *back emf* equation and the *torque* equation for the new operating point.

7. You have now four equations – 2 corresponding to initial operating point and the other 2 corresponding to the new operating point.

8. Take the ratio of the torque equations and the ratio of back emf equations.

9. Solve from the above equations the unknown.

Let us now solve some problems on shunt, separately excited and series motors.

41.2 Shunt motor problems

1. A 220 V shunt motor has armature and field resistance of 0.2 Ω and 220 Ω respectively. The motor is driving a constant load torque and running at 1000 rpm drawing 10 A current from the supply. Calculate the new speed and armature current if an external armature resistance of value 5 Ω is inserted in the armature circuit. Neglect armature reaction and saturation.

Solution

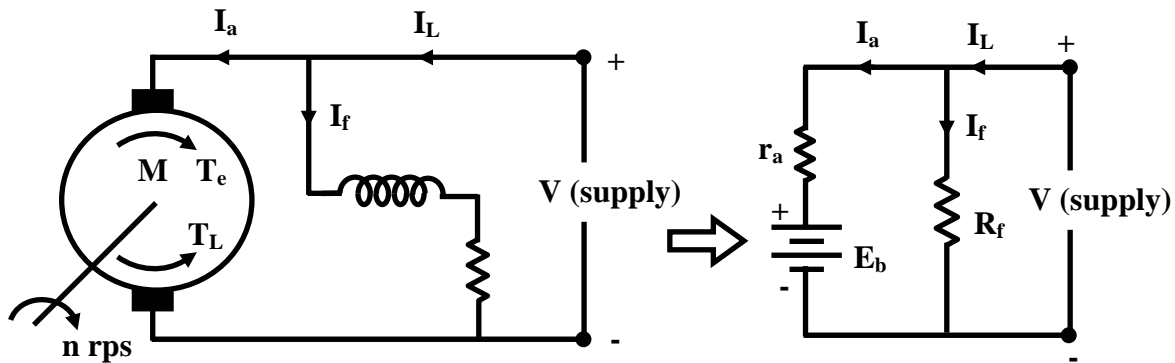The circuit diagram of the shunt motor is:

Figure 41.1: D.C shunt motor.

For initial operating point: $I_{L1} = 10$ A, $r_a = 0.2$ Ω and supply voltage $V = 220$ V.

$$
\begin{aligned}
\text{Field current } I_{f1} &= 220/220 \text{ A} = 1\text{A} \\
\text{Armature current } I_{a1} &= 10\text{A} - 1\text{A} = 9\text{A}
\end{aligned}
$$

Now we write down the expressions for the torque and back emf.

$$
\begin{aligned}
T_{e1} = k_t I_{f1} I_{a1} &= k_t \times 1 \times 9 = T_L \\
E_{b1} = k_g I_{f1} n = k_g \times 1 \times 1000 = V - I_{a1} r_a &= 220 - 9 \times 0.2 = 218.2\text{V} \\
k_g \times 1 \times 1000 &= 218.2\text{V}
\end{aligned}
$$

Since field resistance remains unchanged $I_{f2} = I_{f1} = 1$ A. Let the new steady armature current be I_{a2} and the new speed be n_2. In this new condition the torque and back emf equations are

$$
\begin{aligned}
T_{e2} &= k_t \times 1 \times I_{a2} = T_L \\
E_{b2} &= k_g \times 1 \times n_2 \\
&= V - I_{a2}(r_a + R_{ext}) \\
\therefore k_g \times 1 \times n_2 &= 220 - I_{a2} \times 5.2 \text{ V}
\end{aligned}
$$

Taking the ratios of T_{e2} and T_{e1} we get,

$$
\begin{aligned}
\frac{T_{e2}}{T_{e1}} &= \frac{k_t \times 1 \times I_{a2}}{k_t \times 1 \times 9} \\
\text{Thus, } I_{a2} &= 9 \text{ A}
\end{aligned}
$$

Now taking the ratio emfs $\frac{E_{b2}}{E_{b1}}$, we get,

$$
\frac{k_g \times 1 \times n_2}{k_g \times 1 \times 1000} = \frac{220 - I_{a2} \times 5.2}{218}
$$

$$\frac{n_2}{1000} = \frac{220 - 9 \times 5.2}{218.2}$$

$$\text{or,} \quad \frac{n_2}{1000} = \frac{173.2}{218.2}$$

$$\text{or, } n_2 = \frac{173.2 \times 1000}{218.2}$$

$$\therefore n_2 = 793.76 \text{ rpm}$$

It may be noted that, for constant load torque the steady state armature current does not change with change in the value of the armature resistance.

Let us consider the next problem whose data are similar to the first problem except the fact that load torque is a function of speed.

2. A 220 V shunt motor has armature and field resistances of 0.2 Ω and 220 Ω respectively. The motor is driving load torque, $T_L \propto n^2$ and running at 1000 rpm drawing 10 A current from the supply. Calculate the new speed and armature current if an external armature resistance of value 5 Ω is inserted in the armature circuit. Neglect armature reaction and saturation.

Solution

The field current and armature currents corresponding to the initial operating point are

$$I_{f1} = 1 \text{ A and } I_{a1} = 9 \text{ A}$$

Now torque and back emf equations are:

$$T_{e1} = k_t \times 1 \times 9 = T_{L1}$$
$$E_{b1} = k_g \times 1 \times 1000 = V - I_{a1} r_a = 220 - 9 \times 0.2 = 218.2 \text{V}$$
$$k_g \times 1 \times 1000 = 218.2 \text{V}$$

Let the new steady state armature current be I_{a2} and the new speed be n_2. In this new condition the torque and back emf equations are

$$T_{e2} = k_t \times 1 \times I_{a2} = T_{L2}$$
$$E_{b2} = k_g \times 1 \times n_2$$
$$= V - I_{a2}(r_a + R_{ext})$$
$$\therefore k_g \times 1 \times n_2 = 220 - I_{a2} \times 5.2 \text{ V}$$

Taking the ratios of T_{e2} and T_{e1} we get,

$$\frac{T_{e2}}{T_{e1}} = \frac{T_{L2}}{T_{L1}} = \frac{k_t \times 1 \times I_{a2}}{k_t \times 1 \times 9}$$

$$\frac{n_2^2}{1000^2} = \frac{I_{a2}}{9}$$

$$\frac{I_{a2}}{9} = \frac{n_2^2}{1000^2}$$

or, $\dfrac{n_2}{1000} = \dfrac{\sqrt{I_{a2}}}{3}$

Taking the ratios of E_{b2} and E_{b1} we get,

$$\frac{k_g \times 1 \times n_2}{k_g \times 1 \times 1000} = \frac{220 - I_{a2} \times 5.2}{218.2}$$

$$\frac{n_2}{1000} = \frac{220 - I_{a2} \times 5.2}{218.2}$$

substitution gives: $\dfrac{\sqrt{I_{a2}}}{3} = \dfrac{220 - I_{a2} \times 5.2}{218.2}$

simplification results into the following quadratic equation:

$$0.005 I_{a2}^2 - 1.43\, I_{a2} + 9.15 = 0$$

Solving and neglecting the unrealistic value, I_{a2} = 7 A

$$\therefore n_2 = \frac{\sqrt{I_{a2}}}{3} \times 1000 \text{ rpm}$$

$$= \frac{\sqrt{7}}{3} \times 1000 \text{ rpm}$$

thus, n_2 = 881.9 rpm

3. Initially a d.c shunt motor having r_a = 0.5 Ω and R_f = 220 Ω is running at 1000 rpm drawing 20 A from 220 V supply. If the field resistance is increased by 5%, calculate the new steady state armature current and speed of the motor. Assume the load torque to be constant.

Solution

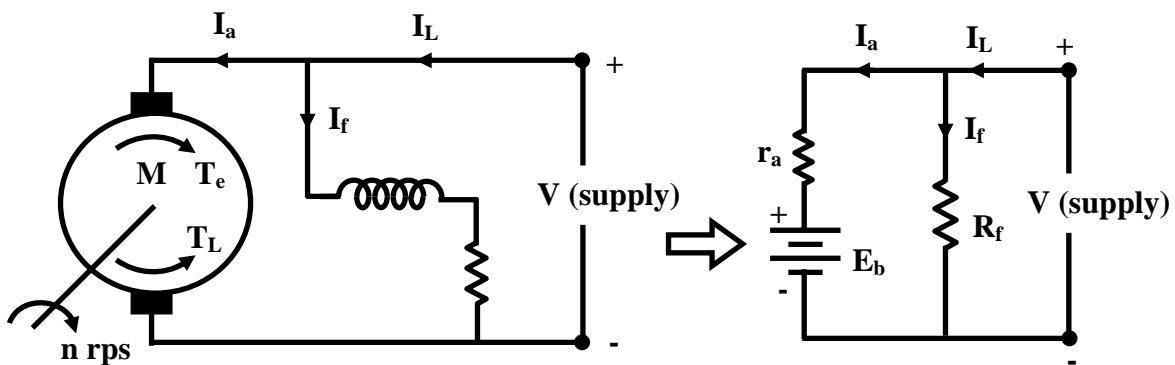As usual let us begin the solution by drawing the shunt motor diagram.

Figure 41.2: D.C shunt motor.

For initial operating point: $I_{L1} = 20$ A, $r_a = 0.5$ Ω and supply voltage $V = 220$ V.

$$\text{Field current } I_{f1} = 220/220 \text{ A} = 1 \text{ A}$$
$$\text{Armature current } I_{a1} = 20 \text{ A} - 1 \text{ A} = 19 \text{ A}$$

Now we write down the expressions for the torque and back emf corresponding to the initial condition.

$$T_{e1} = k_t I_{f1} I_{a2} = k_t \times 1 \times 19 = T_{L1}$$
$$E_{b1} = k_g I_{f1} n_1 = k_g \times 1 \times 1000 = V - I_{a1} r_a = 220 - 19 \times 0.5 = 210.5\text{V}$$
$$k_g \times 1 \times 1000 = 210.5\text{V}$$

In this problem the field Resistance is changed and new value of field resistance is $R_{f2} = 1.05 \times 220 = 231$ Ω. So new field current is $I_{f2} = \frac{220}{231} = 0.95$ A. Let the new steady state armature current be I_{a2} and the new speed be n_2. Since load torque remains constant, we have:

$$T_{e1} = T_{e2}$$
$$k_t I_{f1} I_{a1} = k_t I_{f2} I_{a2}$$
$$\text{or, } I_{f1} I_{a1} = I_{f2} I_{a2}$$
$$\text{or, } 1 \times 19 = 0.95 I_{a2}$$
$$\text{or, } I_{a2} = \frac{19}{0.95}$$
$$\text{or, } I_{a2} = 20 \text{ A}$$

To calculate the new speed n_2, we have to calculate the new back emf:

$$E_{b2} = k_g I_{f2} n_2 = k_g \times 0.95 \times n_2 = 220 - 20 \times 0.5 = 210\text{V}$$
$$k_g \times 0.95 \times n_2 = 210\text{V}$$
$$\therefore \frac{k_g \times 0.95 \times n_2}{k_g \times 1 \times 1000} = \frac{210.5}{210}$$
$$\text{or, } n_2 = \frac{210.5}{210} \times \frac{1000}{0.95}$$
$$\therefore n_2 = 1055.14 \text{ rpm}$$

41.3 Problems on Series Motor

The steps to solve a series motor problem are δ similar to that of solving a shunt motor problem. One has to write down the torque equations and back emf equations corresponding to steady state operating points as before. However, the following points, which distinguishes a series motor from a shunt motor should be carefully noted.

1. Field coil being in series with the armature, in general field current I_f and armature current I_a are same.

2. The field coil resistance r_{se} of a series motor is low and is of the order of armature resistance r_a.

3. The back emf can be calculated as $E_b = V - I_a(r_{se} + r_a)$

4. The back emf E_b is also given by:

$$
\begin{aligned}
E_b &= k\phi n \\
&= k_g I_f n \quad \text{if saturation is neglected.} \\
&= k_g I_a n
\end{aligned}
$$

5. Torque developed by the motor is given by:

$$
\begin{aligned}
T_e &= k\phi I_a \\
&= k' I_f I_a \quad \text{if saturation is neglected.} \\
&= K_t I_a^2
\end{aligned}
$$

6. One should be careful for situations when field current and armature current may not be same. One such situation occurs when a diverter resistance is connected across the field coil for controlling speed.

1. A 220 V d.c series motor has armature and field resistances of 0.15 Ω and 0.10 Ω respectively. It takes a current of 30 A from the supply while running at 1000 rpm. If an external resistance of 1 Ω is inserted in series with the motor, calculate the new steady state armature current and the speed. Assume the load torque remains constant.

Solution

The problem has first been be pictured in the following figure 41.3.
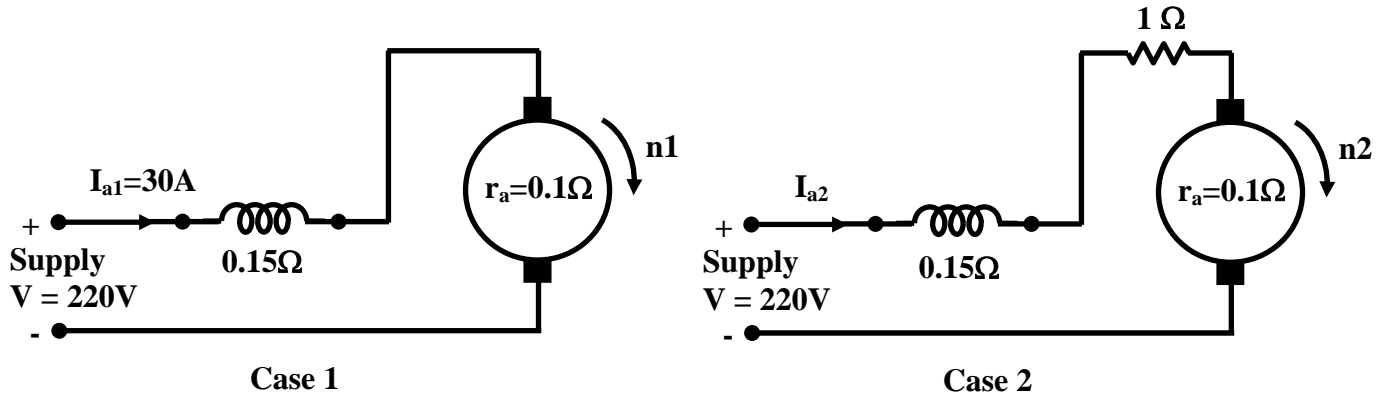

Figure 41.3:

Since the load torque remains constant in both the cases, we have:

$$
\begin{aligned}
T_{e1} = T_{e2} &= T_L \\
\text{or, } K_t I_{a1}^2 &= K_t I_{a2}^2 \\
\text{or, } 30^2 &= I_{a2}^2 \\
\therefore I_{a2} &= 30\text{A}
\end{aligned}
$$

Now equations involving back emfs:

$$
\begin{aligned}
E_{b1} &= V - I_{a1}(r_{se} + r_a) \\
K_g I_{a1} n_1 &= 220 - 30(0.1 + 0.15) \\
K_g 30 \times 1000 &= 212.5 \text{ V}
\end{aligned}
$$

In the second case:

$$
\begin{aligned}
E_{b2} &= V - I_{a2}(r_{se} + r_a + r_{ext}) \\
K_g I_{a2} n_2 &= 220 - 30(0.1 + 0.15 + 1) \\
K_g 30 n_2 &= 182.5 \text{ V}
\end{aligned}
$$

Thus taking the ratio of Eb2 and Eb1 we get:

$$
\begin{aligned}
\frac{k_g \times 30 \times n_2}{k_g \times 30 \times 1000} &= \frac{182.5}{212.5} \\
\therefore n_2 &= \frac{182.5}{212.5} \times 1000 \\
\text{or, } n_2 &= 858.8 \text{ rpm}
\end{aligned}
$$

2. A 220 V d.c series motor has armature and field resistances of 0.15 Ω and 0.10 Ω respectively. It takes a current of 30 A from the supply while running at 1000 rpm. If an external resistance of 1 Ω is inserted in series with the motor, calculate the new steady state armature current and the speed. Assume the load torque is proportional to the square of the speed i.e., $T_L \propto n^2$.

Solution

This problem is same as the first one except for the fact load torque is not constant but proportional to the square of the speed. Thus:

$$
\begin{aligned}
\frac{T_{e2}}{T_{e1}} &= \frac{T_{L2}}{T_{L1}} \\
\frac{T_{e2}}{T_{e1}} &= \frac{n_2^2}{n_1^2} \\
\frac{I_{a2}^2}{I_{a1}^2} &= \frac{n_2^2}{n_1^2} \\
\frac{I_{a2}^2}{30^2} &= \frac{n_2^2}{1000} \\
\text{or, } n_2^2 &= 1.11 \times I_{a2}^2 \\
\therefore n_2 &= 1.05 I_{a2}
\end{aligned}
$$

Ratio of back emfs give:

$$\frac{E_{b2}}{E_{b1}} = \frac{220 - I_{a2} \times 1.25}{212.5}$$

$$\frac{K_g I_{a2} n_2}{K_g I_{a1} n_1} = \frac{220 - I_{a2} \times 1.25}{212.5}$$

$$\frac{K_g I_{a2} n_2}{K_g \times 30 \times n_1} = \frac{220 - I_{a2} \times 1.25}{212.5}$$

$$\frac{I_{a2} \times 1.05 \times I_{a2}}{30 \times 1000} = \frac{220 - I_{a2} \times 1.25}{212.5}$$

$$1.05 I_{a2}^2 = 141.18(220 - I_{a2} \times 1.25)$$

$$1.05 I_{a2}^2 + 176.48 I_{a2} - 31059.6 = 0$$

$$\text{Solving we get, } I_{a2} \approx 107.38\ A$$
$$\text{Hence speed, } n_2 = 1.05 \times I_{a2}$$
$$= 1.05 \times 107.38$$
$$\text{or, } n_2 = 112.75 \text{ rpm.}$$

Next let us solve a problem when a diverter resistance is connected across the field coil.

3. A 220 V d.c series motor has armature and field resistances of 0.15 Ω and 0.10 Ω respectively. It takes a current of 30 A from the supply while running at 1000rpm. If a *diverter* resistance of 0.2 Ω is connected across the field coil of the motor, calculate the new steady state armature current and the speed. Assume the load torque remains constant.

Solution

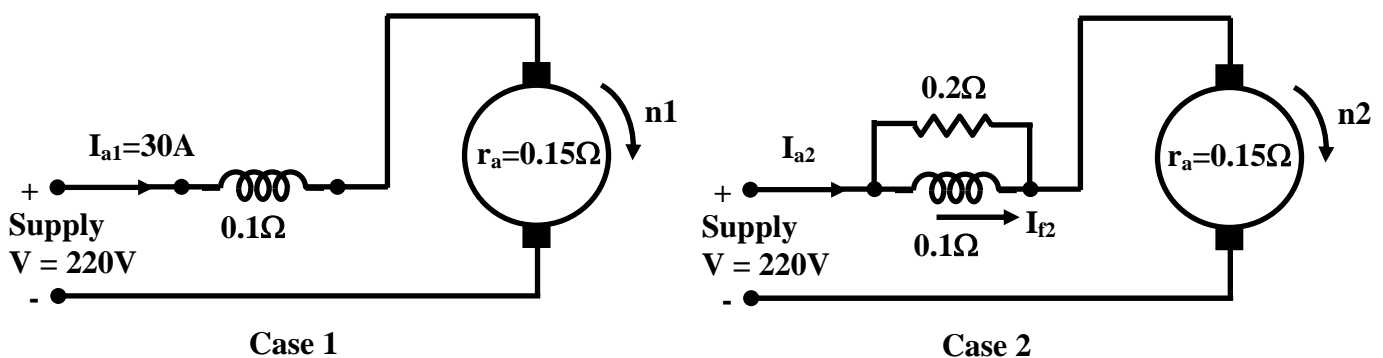Following figure 41.4 shows the 2 cases in which the motor operate.

Figure 41.4:

In the second case it may be noted that $I_{f2} \neq I_{a2}$. In fact, I_{f2} is a fraction of I_{a2}. Since the field coil and diverter are connected in parallel we have:

$$I_{f2} = \frac{R_f}{R_f + R_d} \times I_{a2}$$

$$= \frac{0.1}{0.1+0.2} \times I_{a2}$$

$$\therefore I_{f2} = \frac{1}{3} \times I_{a2}$$

Since load torque remains constant, we have:

$$
\begin{aligned}
T_{e1} = T_{e2} &= T_L \\
\text{or, } K_t I_{a1}^2 &= K_t I_{f2} I_{a2} \\
\text{or, } 30^2 &= I_{f2} I_{a2} \\
\text{or, } 30^2 &= \frac{1}{3} \times I_{a2} \times I_{a2} \\
\text{or, } I_{a2}^2 &= 3 \times 900 \\
\therefore I_{a2} &= 51.96 \text{ A} \\
\text{and } I_{f2} &= 17.32 \text{ A}
\end{aligned}
$$

Now we calculate the back emfs:

$$
\begin{aligned}
E_{b1} &= V - I_{a1}(r_{se}+r_a) \\
K_g I_{a1} n_1 &= 220 - 30(0.1+0.15) \\
K_g\, 30 \times 1000 &= 212.5 \text{ V}
\end{aligned}
$$

In the second case:

$$
\begin{aligned}
E_{b2} &= V - I_{a2}\left(\frac{r_{se} \times r_d}{r_{se}+r_d}+r_a\right) \\
K_g I_{f2} n_2 &= 220 - 51.96\left(\frac{0.1 \times 0.2}{0.1+0.2}+0.15\right) \\
K_g \times 17.32 \times n_2 &= 220 - 51.96(0.67+0.15) \\
K_g \times 17.32 \times n_2 &= 177.39 \text{ V}
\end{aligned}
$$

Thus taking the ratio of Eb2 and Eb1 we get:

$$
\begin{aligned}
\frac{k_g \times 17.32 \times n_2}{k_g \times 30 \times 1000} &= \frac{177.39}{212.5} \\
\therefore n_2 &= \frac{177.39}{212.5} \times 1000 \\
\text{or, } n_2 &= 858.8 \text{ rpm}
\end{aligned}
$$

41.4 D.C generator problems

1. The following data refer to the O.C.C of a D.C separately excited generator at 1000 rpm.

Field current in A:	0.0	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
Armature voltage in V:	5	40	75	100	124	145	162	178	188	195	200	205

The machine is now connected as a *shunt generator* with a total field resistance of 200 Ω and driven at 1000 rpm. Estimate *graphically*: (i) the voltage to which the generator will

build up at no load. (ii) the armature, field and load currents when the terminal voltage is found to be 150 V. Neglect the effect of *armature reaction* and brush drop and assume armature resistance r_a to be 0.8 Ω. (iii) Finally estimate the steady state armature current when the machine terminals are shorted.

Solution

First of all the OCC of the generator is plotted in a graph paper as shown in figure 41.5.
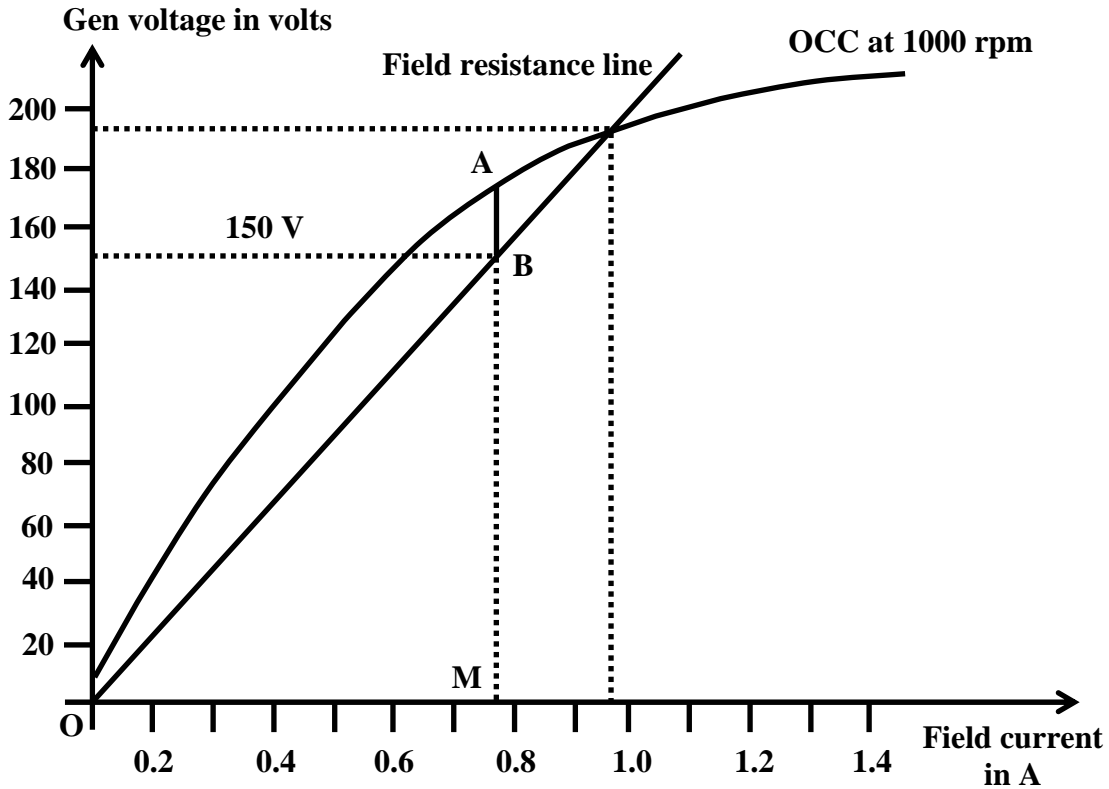

Figure 41.5:

(i) The total field circuit resistance is given to be $R_f = 200$ Ω. The R_f line is now drawn on the same graph paper passing through the origin. The point of intersection of the R_f line and the OCC decides the final no load voltage and can be read from the graph as 192 V and the corresponding field current is 0.96 A.

(ii) Since the terminal voltage V is 150 V(= BM), field current I_f is OM = 0.77 A. Generated voltage E is given by AM. But we know $E = I_a r_a + V$. Hence $I_a r_a = E - V =$ AM − BM = AB. Now from the graph AB = 25 V. $\therefore I_a = 25/0.8 = 31.25$ A. So load current $I_L = I_a - I_f = 31.25 - 0.77 = 30.48$ A. Various currents are shown in the circuit diagram (figure 41.6).

41.5 Operation of d.c machine connected to d.c bus (fixed d.c voltage)

Generally a d.c machine connected to fixed voltage source is expected to operate as a motor drawing current from the source as shown in figure 41.7 (a). However, the same machine may operate as a generator as well feeding power to the d.c bus as shown in figure 41.7 (b). That it

will act as a motor or generator is decided by the fact whether the generated emf E of the machine is *less than* or *more than* the supply voltage V. In figure 41.7 (a), $E < V$ so armature will draw current given by $I_a = (V - E)/r_a$ and the machine acts as motor.
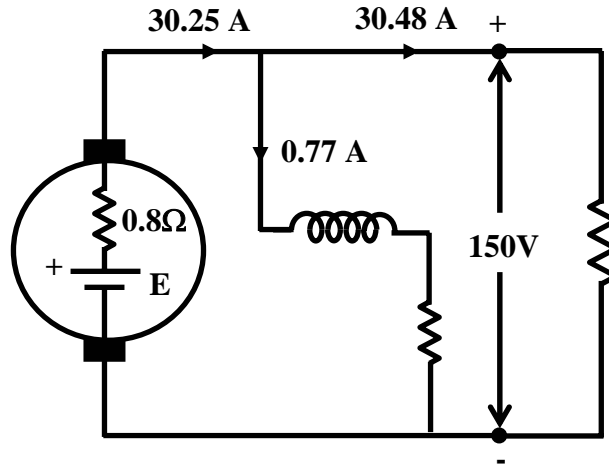

Figure 41.6:

However by some means if E could be made greater than the supply voltage V, the direction of armature current will be reversed and its value is given by $I_a = (E - V)/r_a$ i.e., a current $I_L = I_a - I_f$ will be fed to the supply and the machine will act as a generator. Of course to achieve this, one has to remove the mechanical load from the shaft and run it at higher speed with a prime mover to ensure $E > V$. Remember E being equal to $k_g I_f n$ and I_f held constant, one has to increase speed so as to make E more than V. The following problem explains the above operation,
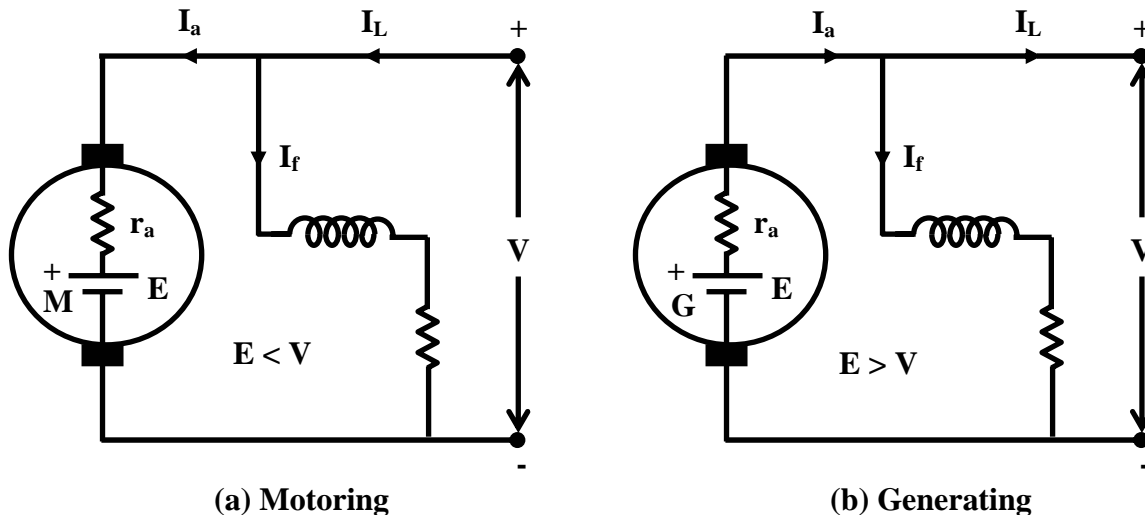

(a) Motoring   **(b) Generating**

Figure 41.7:

2. A 200 V, d.c shunt machine has an armature resistance of 0.5 Ω and field resistance of 200 Ω. The machine is running at 1000 rpm as a motor drawing 31 A from the supply mains. Calculate the speed at which the machine must be driven to achieve this as generator.

Solution

First draw the circuit diagrams showing the motoring and generating mode of operation indicating the currents and their directions as in figure 41.8.

For motor mode:

Current drawn from supply, I_L = 31 A

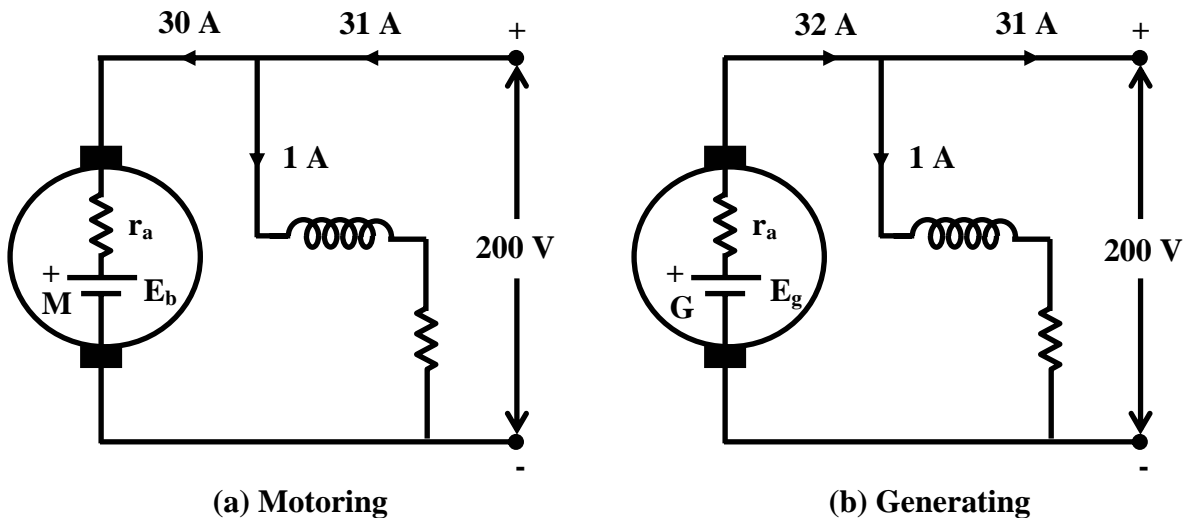**(a) Motoring**   **(b) Generating**

Figure 41.8:

$$
\begin{aligned}
\text{Field current, } I_f &= 200/200 = 1 \text{ A} \\
\text{Armature current, } I_{a1} &= 31 - 1 = 30 \text{ A} \\
\text{Back emf, } E_b &= 200 - 30 \times 0.5 \\
\text{or, } kI_f n_1 &= 185 \text{ V} \\
\text{Since speed, } n_1 &= 1000 \ rpm \\
\text{so, } k \times 1 \times 1000 &= 185 \text{ V}
\end{aligned}
$$

Similarly for generator mode:

$$
\begin{aligned}
\text{Current fed to supply, } I_L &= 31 \text{ A} \\
\text{Field current, } I_f &= 200/200 = 1 \text{ A} \\
\text{Armature, } I_{a2} &= 31 + 1 = 32 \text{ A} \\
\text{Generated emf, } E_g &= 200 + 32 \times 0.5 \\
\text{or, } k I_f n_2 &= 216\text{V where } n_2 \text{ is the unknown speed}
\end{aligned}
$$

Now taking the ratio of E_g and E_b we get:

$$
\frac{E_g}{E_b} = \frac{k \times 1 \times n_2}{k \times 1 \times 1000} = \frac{216}{185}
$$

$$
n_2 = \frac{216}{185} \times 1000 = 1167.5 \text{ rpm}
$$